Hilbert's Method of Analogy: Signs and Axiomatics in Physics and Mathematics

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There is a marked contrast between Emil du Bois-Reymond's pessimism about solving the problems of epistemology in the Ignorabimus (we will not know) lecture of 1872, and Hilbert's determination – "In mathematics there is no ignorabimus" – in his Paris lecture of 1900. A well-known source of Hilbert's optimism is his engagement with Dedekind's and Riemann's axiomatic method in mathematics. However, careful attention to Hilbert's examples and terminology, from his Paris lecture to his 1918 "Axiomatisches Denken," reveals another, illuminating source of Hilbert's axiomatic method: Heinrich Hertz's Bildtheorie in mechanics, which has its origin in Hermann von Helmholtz's sign theory. In the 1894 Principles of Mechanics, Hertz argues for a physical analogue of Dedekind's and Riemann's axiomatic methods in mathematics: that the theorems of mechanics should be deduced formally from postulated mechanical principles, without further appeal to experience.

I argue that reading Hilbert in this context allows for the resolution of significant problems for Hilbert's epistemology. Kitcher (1976) describes several intriguing puzzles for Hilbert's epistemology and axiomatics—in particular, what makes the statements of mathematics true? Do the terms of Hilbert's mathematics refer to objects? Kitcher reads Hilbert's epistemology as based on Kantian pure intuition, as beginning with concrete observation but then following abstract a priori rules, the axioms in Hilbert's case. But, Kitcher points out, this does not explain why Hilbert's symbols should refer to objects, and thus why Hilbert's methods should not be, as Weyl puts it in the 1920s, a mere formal game with symbols.

But Hertz's Bildtheorie and Helmholtz's sign theory take the terms of even physical theories to symbolize their objects, not to resemble them in any feature. In his 1900 and 1918 lectures, Hilbert adapts Helmholtz's and Hertz's symbolic reading of physical signs to mathematical signs, concluding that mathematical signs such as strokes do not necessarily resemble their objects. On this reading, the key question for Hilbert's epistemology is not whether mathematical terms resemble real objects, but rather, how we establish that a given mathematical proof is truth-tracking. Hertz developed an axiomatic treatment of physical theorems: that a theorem has a warrant if it can be deduced from a postulated physical law, without further ad hoc appeal to experience. Hilbert innovates over Hertz between 1900 and 1918, by applying Hertz's axiomatic method for mechanics to mathematical systems including ideal elements, and by arguing that the solutions to mathematical and physical problems are interrelated. This allows Hilbert to develop an analogy between physical and mathematical truth, which yields an epistemology that is more innovative and more interesting than is usually appreciated. I briefly examine an example of Hilbert's method, of drawing warranted analogies between physical and mathematical problems, and conclude by sketching a way this method could be applied profitably in a contemporary context.